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EXPERIMENTAL DETERMINATION OF THE HYDRODYNAMIC MASS OF VARIOUS --ETC(U)
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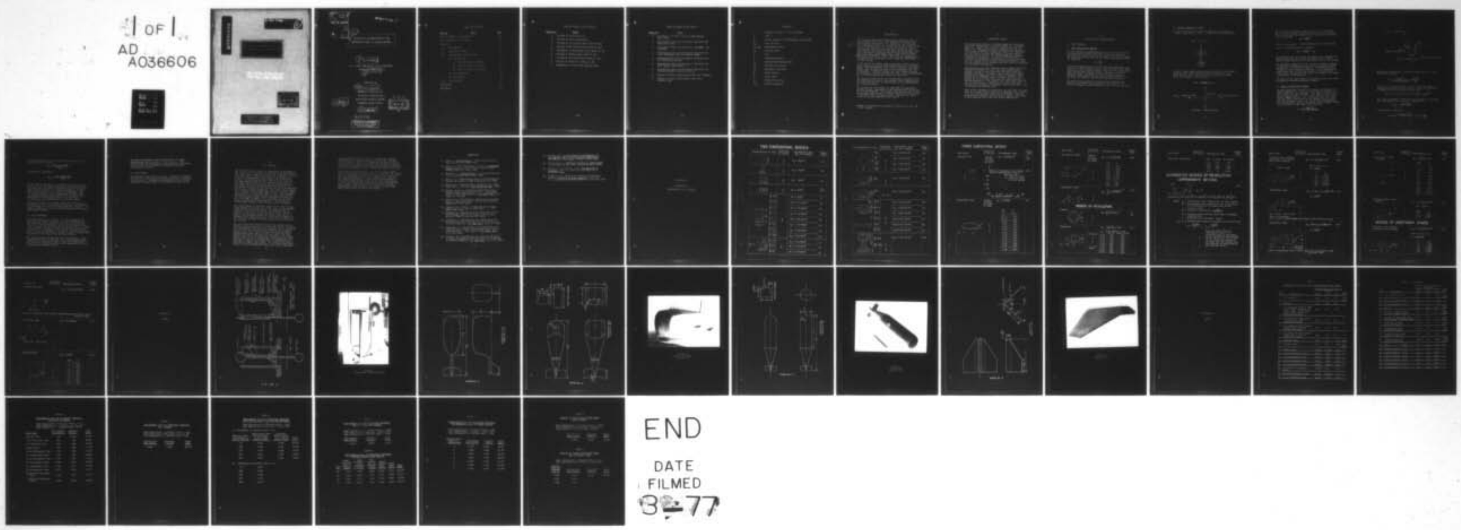

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MOST Project -4

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EXPERIMENTAL DETERMINATION OF THE HYDRODYNAMIC MASS OF VARIOUS BODIES

MOST Project -4

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EXPERIMENTAL DETERMINATION OF THE HYDRODYNAMIC MASS OF VARIOUS BODIES.

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Final Report, On

U. S. NAVAL UNDERWATER SOUND LABORATORY

CONTRACT N70024-1024

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Submitted by

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March 1965

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NOTATION

c	damping constant or speed of sound
j	$\sqrt{-1}$
K	spring constant or hydrodynamic mass factor
m_b	mass of body
m_h, M_h	hydrodynamic mass
m_t	apparent mass
t	time
x	body displacement
x_m	maximum body displacement
X_m	mechanical reactance
Z_m	mechanical impedance
ρ	fluid density
ϕ	phase angle
ω	angular frequency
ω_n	natural frequency

I

INTRODUCTION

If a constant mass body immersed in a fluid is given an acceleration and if the force required to produce the acceleration is measured, one finds that the measured requires a force is greater than the product of the mass of the body and the acceleration. The reason for this is that in accelerating the body one must also accelerate some of the fluid in order that it may move out of the way. The body, therefore, appears to have a mass that is larger than its actual mass. This larger mass is called the "apparent mass" of the body, and it may be considered as the sum of the actual mass of the body and the "hydrodynamic mass."

In the design of towing cables and other towing gear, it is necessary to know the hydrodynamic mass of the towed body. The hydrodynamic mass can be calculated for simple shapes such as spheres, disks, and ellipsoids, but the calculation for winged bodies and other complex shapes is difficult. Little experimental data on the hydrodynamic mass of three-dimensional bodies is available.

The object of this study was to determine experimentally the hydrodynamic mass of streamlined towed bodies and to determine the effect of oscillations on the hydrodynamic mass of the bodies.

The results of this study are summarized in Appendix A of this report. Appendix A also includes the results of other investigators. The actual data obtained are on file on punched cards in the Mechanical Engineering Department at the University of Rhode Island. The data are also available in the work by Patton (13).*

*Numbers in parentheses designate references at the end of the report.

II

LITERATURE SURVEY

The theoretical aspects of hydrodynamic mass have been discussed extensively by Lamb (1), Munk (2), and Birkhoff (3). Lamb's book contains exact values for the sphere; the circular cylinder of infinite length; the flat strip of infinite length; the disc; the ellipsoid; and the sphere in close proximity to another sphere. Munk (2) has calculated the hydrodynamic mass for the elliptical flat plate. An excellent discussion is presented by Birkhoff(3) utilizing the tensor notation and the concept of an inertial Lagrangian system. Zahm (4) has calculated the hydrodynamic mass for an ellipsoid of revolution in rotation.

Hydrodynamic masses for two-dimensional bodies have been computed by Wendel (5) using the Schwartz-Christofel method. Bryson (6) has extended this work using the hodograph method. A complete expression of the hydrodynamic mass components for a body of arbitrary shape moving in an arbitrary manner is given by Imlay (7). Darwin (8) has recently suggested the drift concept of hydrodynamic mass. The problem of a body oscillating in the free surface of a liquid has received extensive theoretical treatment by Landweber and Macagno (10), (11) and Macagno and Macagno (12).

Most of the experimental work that has been done has been done in Germany and is discussed by Brahmig (7). Practically all of this work, however, is concerned with the mathematically simple shapes, such as spheres and discs. A small portion has to do with marine propellers.

III

EXPERIMENTAL INVESTIGATION

A. TEST METHODS

1. Free Translation Method

In this method the hydrodynamic mass of an immersed body is determined by giving the body an acceleration and measuring the force required to produce the acceleration. The apparent mass of the body can then be determined from the equation

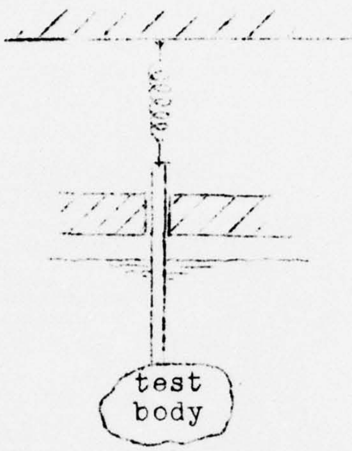
$$m_t = \frac{F}{\ddot{x}}$$

and the hydrodynamic mass is obtained by subtracting the actual mass of the body from m_t. The force F in the above equation is the resultant force acting on the accelerated body and includes viscous forces. Because a certain amount of time is required for the viscous forces to build up, this method should yield reliable results if the data are taken at the instant the motion starts.

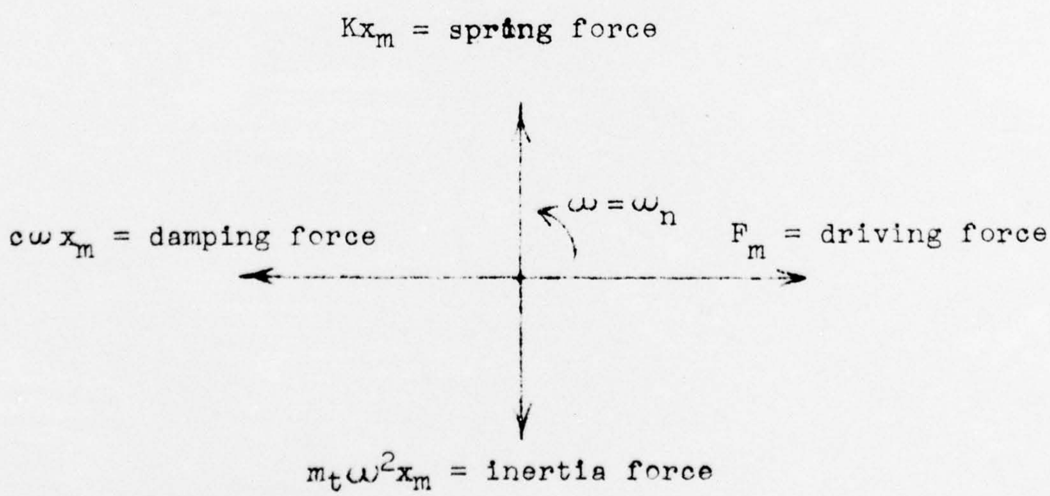
The first tests performed in this study utilized this method. The method was discontinued in favor of the natural frequency method which gave more reliable results.

2. Natural Frequency Method

In this method the test body is mounted as the mass in a simple spring-mass system as shown in the sketch below.



If this simple spring-mass system is driven at its natural frequency the forces involved will be related to each other in the manner shown in the vector diagram below. (See, for example, reference 14.)



At resonance the driving force balances the frictional force and the inertia force balances the spring force. From the inertia force-spring force equality one obtains the relationship

$$m_t = K/\omega_n^2$$

Since $m_t = m_b + m_b$ the hydrodynamic mass of the body can be calculated from the equation

$$m_h = (K/\omega_n^2) - m_b$$

The hydrodynamic mass can be determined by measuring the spring constant, the natural frequency of the system with the body immersed, and the actual mass of the system.

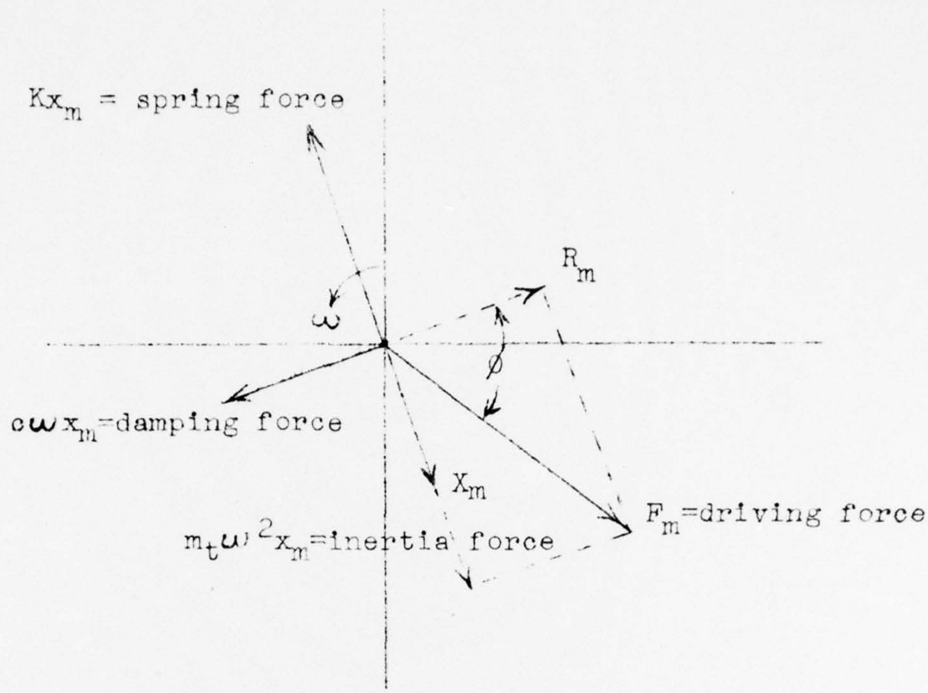
The principal advantage of the natural frequency method is that the hydrodynamic mass can be determined without considering the effect of damping. The principal disadvantage of the method is that for a given spring-mass combination there is but one frequency; consequently, a large number of springs and masses are required to obtain a wide range of frequencies.

The bulk of the experimental work performed in this study utilized the natural frequency method.

3. Forced Oscillation Method

In this method the test body is mounted as the mass in a simple spring-mass system and the system is driven at a given frequency and amplitude. The forces involved are related to each other in the manner shown in the vector diagram **which** follows. (See reference 14.) The angle $\emptyset$ is the phase angle between the driving force vector and the velocity vector. The steady state solution of the differential equation of motion for the driven system can be written in the form (reference 15)

$$x = \frac{-jFe^{j\omega t}}{\omega c + j(\omega^2 t - K)}$$



Differentiation of this expression with respect to time yields the velocity

$$\dot{x} = \frac{Fe^{j\omega t}}{c + j(\omega m_t - K/\omega)} = \frac{Fe^{j\omega t}}{Z_m}$$

where $Z_m = c + j(\omega m_t - K/\omega)$ and is called the complex mechanical impedance of the system. This expression for the impedance can be written in the form

$$Z_m = c + j(\omega m_t - K/\omega) = c + jX_m = |Z_m| e^{j\phi}$$

where the mechanical reactance X_m is defined as $\omega m_t - K/\omega$. The magnitude of the mechanical impedance is

$$|Z_m| = \sqrt{c^2 + X_m^2}$$

and its phase angle ϕ is

$$\phi = \tan^{-1} \frac{\omega m_t - K/\omega}{c} = \tan^{-1} \frac{X_m}{c}$$

Differentiation of the velocity with respect to time yields the acceleration

$$\ddot{x} = \frac{-\omega F \sin(\omega t - \emptyset)}{|Z_m|}$$

Solving for $|Z_m|$ gives

$$|Z_m| = \frac{-\omega F \sin(\omega t - \emptyset)}{\ddot{x}}$$

In the actual experiment a simultaneous record of the force, F, and the acceleration, $\ddot{x}$, as functions of time was made on a two channel recorder while the immersed body was being driven at a given frequency ω. From this record the phase angle $\emptyset$ can be determined and the above equation allows computation of $|Z_m|$. From the reactive component of $|Z_m|$ one can calculate m_t. Subtraction of the actual mass of the body from m_t yields m_h, the hydrodynamic mass of the body.

The forced oscillation method allows determination of hydrodynamic mass at any desired frequency and amplitude. A small number of tests were run using this method. Further studies are being conducted utilizing the forced oscillation method.

B. TEST APPARATUS

A one-sixteenth scale drawing and a photograph of the apparatus are shown in Figures 1 and 2 of Appendix B. The apparatus was designed to allow it to be used for either the natural frequency method or the forced oscillation method. If used for forced oscillations the apparatus provided a frequency from zero to two cycles per second up to an amplitude of 6 inches. Detailed drawings of the apparatus are on file in the Mechanical Engineering Department at the University of Rhode Island.

The forces exerted on the body were measured by a load sensor which was a 1-1/16-inch O.D. nylon cylinder with 1/32-inch wall thickness with a four gage SR-4 bridge. The system was therefore temperature compensated and not influenced by bending loads.

The body acceleration was determined by a Giannini accelerometer with a range of 0 to 2 1/2 g. The signals from the accelerometer and from the load sensor were recorded on a Baldwin-Lima-Hamilton Meterite two-channel recorder.

C. TEST BODIES

Thirty-three test bodies were used. These are described in terms of certain characteristic dimensions in Table 1 of Appendix C. Drawings and photographs of the "towed" bodies are shown in Figures 3 through 9 of Appendix B.

IV

TEST RESULTS

The results of this study are summarized in Appendix A. The results of the various individual tests from which Appendix A was compiled are tabulated in Tables 2 through 9 in Appendix C. The results have been tabulated in terms of a hydrodynamic mass factor and an impedance factor. The hydrodynamic mass factor is the ratio of the hydrodynamic mass to some characteristic mass of the body, usually the mass of the water displaced by the body. The impedance factor is the ratio of the mechanical impedance to the product of the characteristic mass and the angular frequency. The phase angle listed in all tables is the angle between the resistance (damping) component and the impedance vector. The impedance values have been corrected for the fluid friction on the model support shaft and the mechanical friction in the test frame assembly. The resistance of the test frame assembly was determined by the log decrement method after running tests without a model attached.

The displacement to diameter ratio is the ratio of the total distance travelled in a half cycle to the minimum horizontal length of the body. The submergence to diameter ratio is the ratio of the distance from the surface of the water to the geometric center of the body to the minimum horizontal length of the body. The dimensionless frequency is the ratio $\omega A/c$ in which c, the velocity of sound in water, was taken as 5000 feet per second.

The data runs most likely to have large errors for the natural frequency method of testing were those for which the hydrodynamic mass was small compared to the combined mass of the body and shaft and for which the resonant frequency was low. The usual data run yielded hydrodynamic mass values with an error of ± 3%. The most significant factor affecting the accuracy of the tests was the size of the testing tank. Corrections for tank boundary effects were accomplished by calibrating the tank with a sphere and a disc. The calibration correction for the sphere was applied to all three-dimensional bodies and the calibration correction for the disc was

applied to all flat plate bodies; hence, the boundary corrections were only approximate for nearly all the bodies tested. However, the hydrodynamic masses of the elliptical flat plates were within 4% of the theoretical value after the above calibration correction was applied, so it was concluded that the results are quite reliable.

The results listed in Appendix A for the three-dimensional bodies, with the exception of those for the sphere, and the ellipsoid, are direct results of this study and were not available before it was undertaken. Studies are continuing using the forced oscillation method in order to definitely determine the effects of frequency of oscillation and depth of submergence on hydrodynamic mass. These continuing studies will also determine hydrodynamic mass and impedance factors for the towed body shapes in the direction of towing.

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13. Patton, K.T., An Experimental Determination of Hydrodynamic Masses and Mechanical Impedences, M.S. Thesis, University of Rhode Island, 1965.

14. Timoshenko, S., Vibration Problems in Engineering, D. van Nostrand Co., Inc., Princeton, N.J., 1937.

15. Kinsler, L.E. and Frey, A.R., Fundamentals of Acoustics, 2nd. ed., John Wiley & Sons, Inc., New York, 1962.

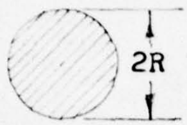
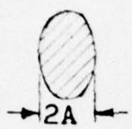
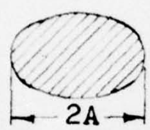
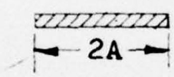
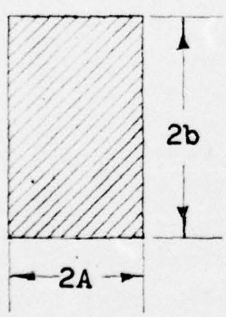
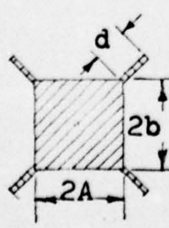
16. Landweber, L., "Motion of Immersed and Floating Bodies," Handbook of Fluid Dynamics, edited by V. L. Streeter, McGraw-Hill Book Co., New York, 1961.

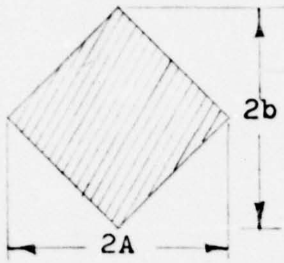
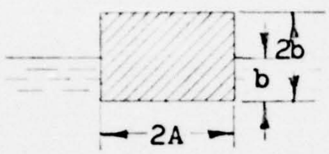
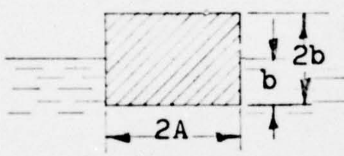
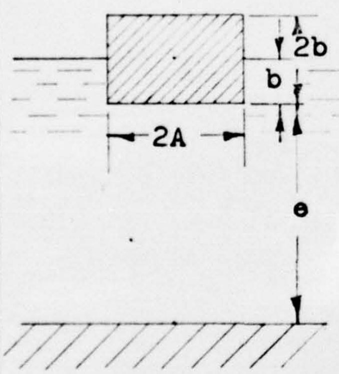
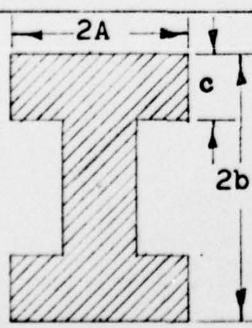
APPENDIX A

A SUMMARY OF

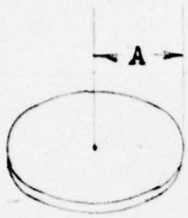
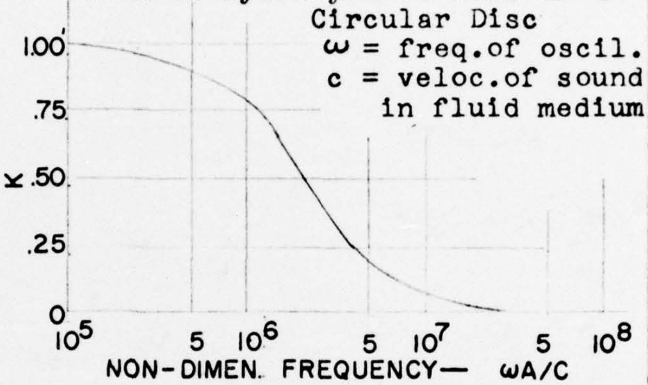
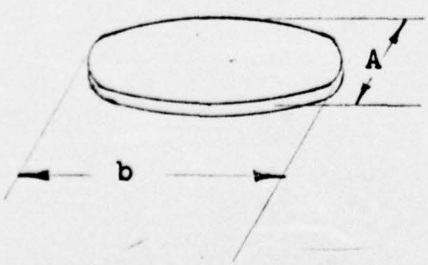
HYDRODYNAMIC MASS FACTORS

TWO DIMENSIONAL BODIES

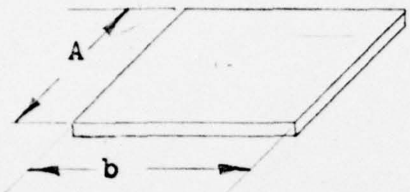
Cross-Section of Body		Direction of Motion	Hydrodynamic Mass per Unit Length	Reference
		↕	$M_h = \pi \rho R^2$	(4)
		↕	$M_h = \pi \rho A^2$	(4)
		↕	$M_h = \pi \rho A^2$	(4)
		↕	$M_h = \pi \rho A^2$	(4), (13)
	$\frac{A}{b} = \infty$	↕	$M_h = \pi \rho A^2$	(4)
	$\frac{A}{b} = 10$		$M_h = 1.14 \pi \rho A^2$	(4)
	$\frac{A}{b} = 5$		$M_h = 1.21 \pi \rho A^2$	(4)
	$\frac{A}{b} = 2$		$M_h = 1.36 \pi \rho A^2$	(4)
	$\frac{A}{b} = 1$		$M_h = 1.51 \pi \rho A^2$	(4)
	$\frac{A}{b} = \frac{1}{2}$		$M_h = 1.70 \pi \rho A^2$	(4)
	$\frac{A}{b} = \frac{1}{5}$		$M_h = 1.98 \pi \rho A^2$	(4)
	$\frac{A}{b} = \frac{1}{10}$		$M_h = 2.23 \pi \rho A^2$	(4)
$\frac{A}{b} = 1$ 	$\frac{d}{A} = .05$	↕	$M_h = 1.61 \pi \rho A^2$	(4)
	$\frac{d}{A} = .10$		$M_h = 1.72 \pi \rho A^2$	(4)
	$\frac{d}{A} = .25$		$M_h = 2.19 \pi \rho A^2$	(4)

Cross-Section of Body	Direction of Motion	Hydrodynamic Mass per Unit Length	Reference
 $\frac{A}{b}=2$	↕	$M_h = 0.85\,\pi\rho A^2$	(4)
$\frac{A}{b}=1$		$M_h = 0.76\,\pi\rho A^2$	(4)
$\frac{A}{b}=\frac{1}{2}$		$M_h = 0.67\,\pi\rho A^2$	(4)
$\frac{A}{b}=\frac{1}{5}$		$M_h = 0.61\,\pi\rho A^2$	(4)
 $\frac{A}{b}=1$	↕	$M_h = 0.75\,\pi\rho A^2$	(4)
 $\frac{A}{b}=1$	↔	$M_h = 0.25\,\pi\rho A^2$	(4)
 $\frac{A}{b}=1,\ \frac{e}{b}=\infty$	↕	$M_h = 0.75\,\pi\rho A^2$	(4)
$\frac{e}{b}=2.6$		$M_h = 0.83\,\pi\rho A^2$	(4)
$\frac{e}{b}=1.8$		$M_h = 0.89\,\pi\rho A^2$	(4)
$\frac{e}{b}=1.5$		$M_h \approx 1.00\,\pi\rho A^2$	(4)
$\frac{e}{b}=.5$		$M_h \approx 1.35\,\pi\rho A^2$	(4)
$\frac{e}{b}=.25$		$M_h \approx 2.00\,\pi\rho A^2$	(4)
 $\frac{A}{b}=.572$ $\frac{A}{c}=1.333$	↕	$M_h = 2.11\,\pi\rho A^2$	(13)

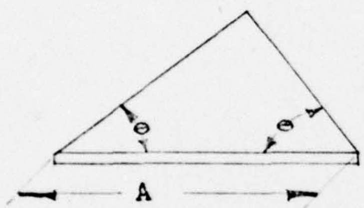
THREE DIMENSIONAL BODIES

Body Shape	Direction of Motion	Hydrodynamic Mass	Reference
Circular Disc 	Perpendicular to plane of disc	$M_h = K(8/3)\rho A^3$	(1) (15)
		Effect of Frequency of Oscillation on Hydrodynamic Mass of a Circular Disc 	(15)
Elliptical Disc 	Perpendicular to plane of disc	$M_h = KbA^2\frac{\pi}{6}\rho$	(2)

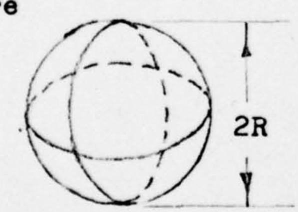
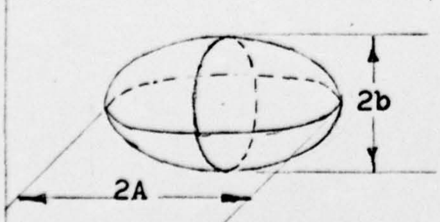
b/A	K
∞	1.00
14.3	.991
12.75	.987
10.43	.985
9.57	.983
8.19	.978
7.00	.972
6.00	.964
5.02	.952
4.00	.933
3.00	.900
2.00	.826
1.50	.748
1.00	.637

Body Shape	Direction of Motion	Hydrodynamic Mass	Reference
Rectangular Plate 	Perpendicular to plane of plate	$M_h = K \pi \rho \left(A^2/4\right) b$	(13)

b/A	K
1.0	.478
1.5	.680
2.0	.840
2.5	.953
3.0	1.00
3.5	1.00
4.0	1.00
∞	1.00

Body Shape	Direction of Motion	Hydrodynamic Mass	Reference
Triangular Plate 		$M_h = \frac{\rho}{3} A^3 \left(\frac{\tan \theta}{\pi}\right)^{3/2}$	(13)

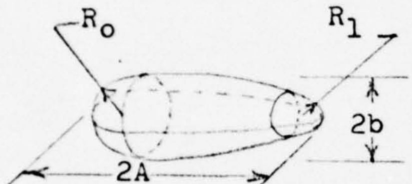
BODIES OF REVOLUTION

Body Shape	Direction of Motion	Hydrodynamic Mass	Reference
Sphere 		$M_h = \left(2/3\right) \pi \rho R^3$	(1) (3)
Ellipsoid 	Vertical / Axial	$M_h = K\left(4/3\right) \pi \rho A b^2$	(1)

A/b	K for Axial Motion	K for Vert. Motion
1.00	.500	.500
1.50	.305	.621
2.00	.209	.702
2.51	.156	.763
2.99	.122	.803
3.99	.082	.860
4.99	.059	.895
6.01	.045	.918
6.97	.036	.933

Body Shape	Direction of Motion	Hydrodynamic Mass			Reference
Ellipsoid (continued)		A/b	K Axial	K Lateral	
		8.01	.029	.945	
		9.02	.024	.954	
		9.97	.021	.960	
		∞	0	1.000	

ELONGATED BODIES OF REVOLUTION
(APPROXIMATE METHOD)



$\leftrightarrow$

(16)

$$M_h = K_1 \rho V = K_e \left[1 + 17.0\left(C_p - \frac{2}{3}\right)^2 + 2.49\left(m - \frac{1}{2}\right)^2 + 0.283\left[\left(r_o - \frac{1}{2}\right)^2 + \left(r_1 - \frac{1}{2}\right)^2\right]\right]\rho V$$

where: K_1 = Hydrodynamic Mass Coefficient for Axial Motion

K_e = Hydrodynamic Mass Coefficient for Axial Motion of an ellipsoid of the same ratio of A/b

V = Volume of body

C_p = Prismatic Coefficient = $\frac{4V}{\pi b^2(2A)}$

L_m = Distance along body axis from nose to maximum cross-section

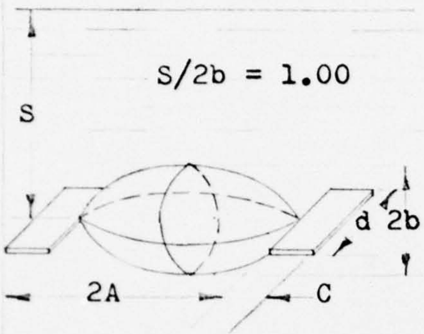
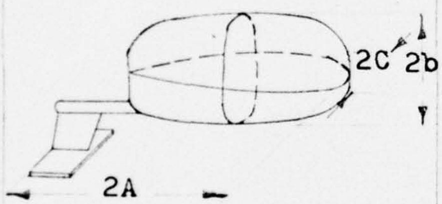
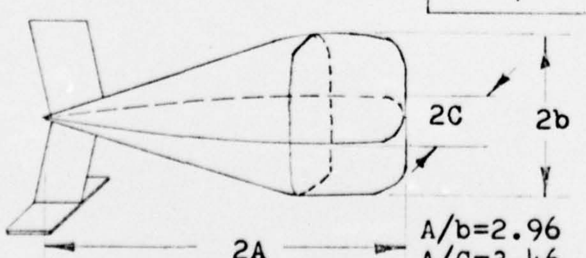
M = Nondimensional Abscissa = $L_m/2A$

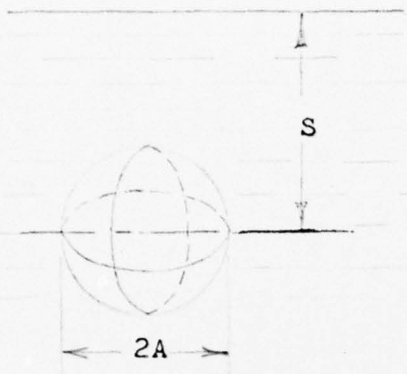
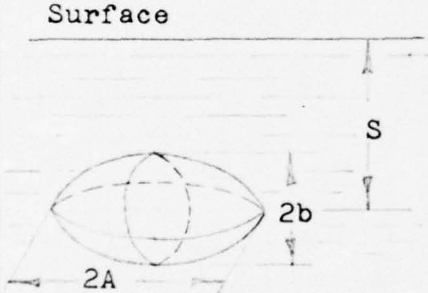
r_o, r_1 = Dimensionless radii of curvature at nose and tail

$$r_o = \frac{R_o(2A)}{b^2} \quad ; \quad r_1 = \frac{R_1(2A)}{b^2}$$

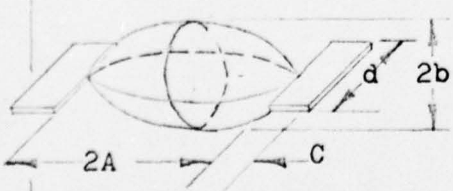
$\updownarrow$

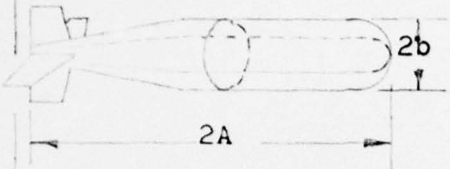
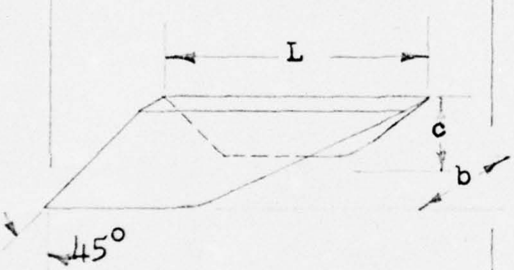
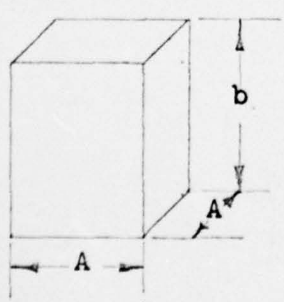
Munk has shown that the hydrodynamic mass of an elongated body of revolution can be reasonably approximated by the product of the density of the fluid, the volume of the body and the K-factor for an ellipsoid of revolution of the same A/b ratio.

Body Shape	Direction of Motion	Hydrodynamic Mass	Reference
Ellipsoid with Attached Rectangular Flat Plates Near a Free Surface  	↕	$M_h = K \cdot (4/3)\pi \rho Ab^2$ $A/b = 2.00$; $C = b$ $N = Cd/\pi Ab$ N — K 0 — .9130 .20 — 1.0354 .30 — 1.3010 .40 — 1.4610 .50 — 1.5706	(13)
Streamlined Body  $A/b = 2.38$ $A/C = 2.11$ Area of Horizontal Tail = 25% of Body Maximum Horizontal Cross Sectional Area	↕	$M_h = 1.124\ \rho (4/3)(\pi Ad^2)$ $d = \frac{C + b}{2}$	(13)
Streamlined Body  $A/b=2.96$ $A/C=3.46$ Area of Horizontal Tail = 20% of Body Max. Horizontal Cross Sectional Area	↕	$M_h = 0.672\ \rho (4/3)(\pi Ad^2)$ $d = \frac{C + b}{2}$	(13)

Body Shape	Direction of Motion	Hydrodynamic Mass	Reference
Sphere Near A Free Surface 	↕	$M_h = K(2/3)\pi\rho A^3$ S/2A — K 0 — .50 .5 — .88 1.0 — 1.08 1.5 — 1.16 2.0 — 1.18 2.5 — 1.18 3.0 — 1.16 3.5 — 1.12 4.0 — 1.04 4.5 — ≈1.00	(13)
Ellipsoid Near A Free Surface 	↕	$M_h = K\ (4/3)\pi\rho\ Ab^2$ $A/b = 2.00$ S/2b — K 1.00 — .913 2.00 — .905	(13)

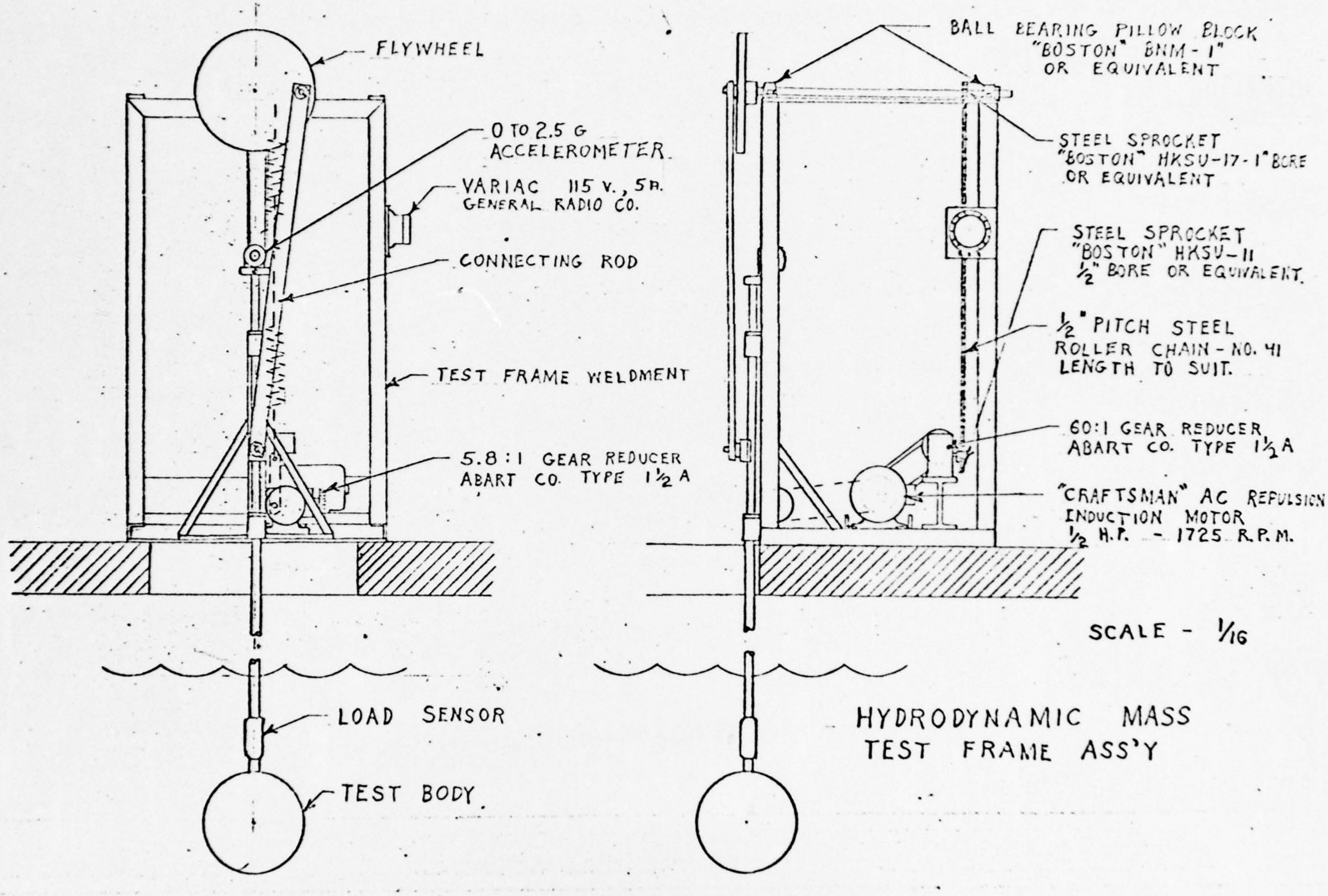
BODIES OF ARBITRARY SHAPE

Body Shape	Direction of Motion	Hydrodynamic Mass	Reference
Ellipsoid with Attached Rectangular Flat Plates 	↕	$M_h = K\ (4/3)\pi\rho\ Ab^2$ $A/b = 2.00;\ C = b$ $N = Cd/(\pi Ab)$ N — K 0 — .7024 .20 — .8150 .30 — 1.0240 .40 — 1.1500 .50 — 1.2370	(13)

<table>
<tr><th>BODY SHAPE</th><th>Direction of Motion</th><th>Hydrodynamic Mass</th><th>Reference</th></tr>
<tr><td>Torpedo Type Body

$A/b = 5.0$
Area of Horizontal Tail = 10% of Body Maximum Horizontal Cross-Sectional Area</td><td></td><td>$M_h = 0.818\,\pi\rho b^2(2A)$</td><td>(13)</td></tr>
<tr><td>V-Fin Type Body

$L/b = 1.0$ $L/c = 2.0$</td><td></td><td>$M_h = 0.3975\rho L^3$</td><td>(13)</td></tr>
<tr><td>Parallelepiped
</td><td></td><td>$M_h = K\rho A^2 b$
<table><tr><th>b/A</th><th>K</th></tr><tr><td>1</td><td>2.32</td></tr><tr><td>2</td><td>.86</td></tr><tr><td>3</td><td>.62</td></tr><tr><td>4</td><td>.47</td></tr><tr><td>5</td><td>.37</td></tr><tr><td>6</td><td>.29</td></tr><tr><td>7</td><td>.22</td></tr><tr><td>10</td><td>.10</td></tr></table></td><td>(13)</td></tr>
</table>

APPENDIX B

FIGURES



HYDRODYNAMIC MASS
TEST FRAME ASS'Y

FIG. No 1

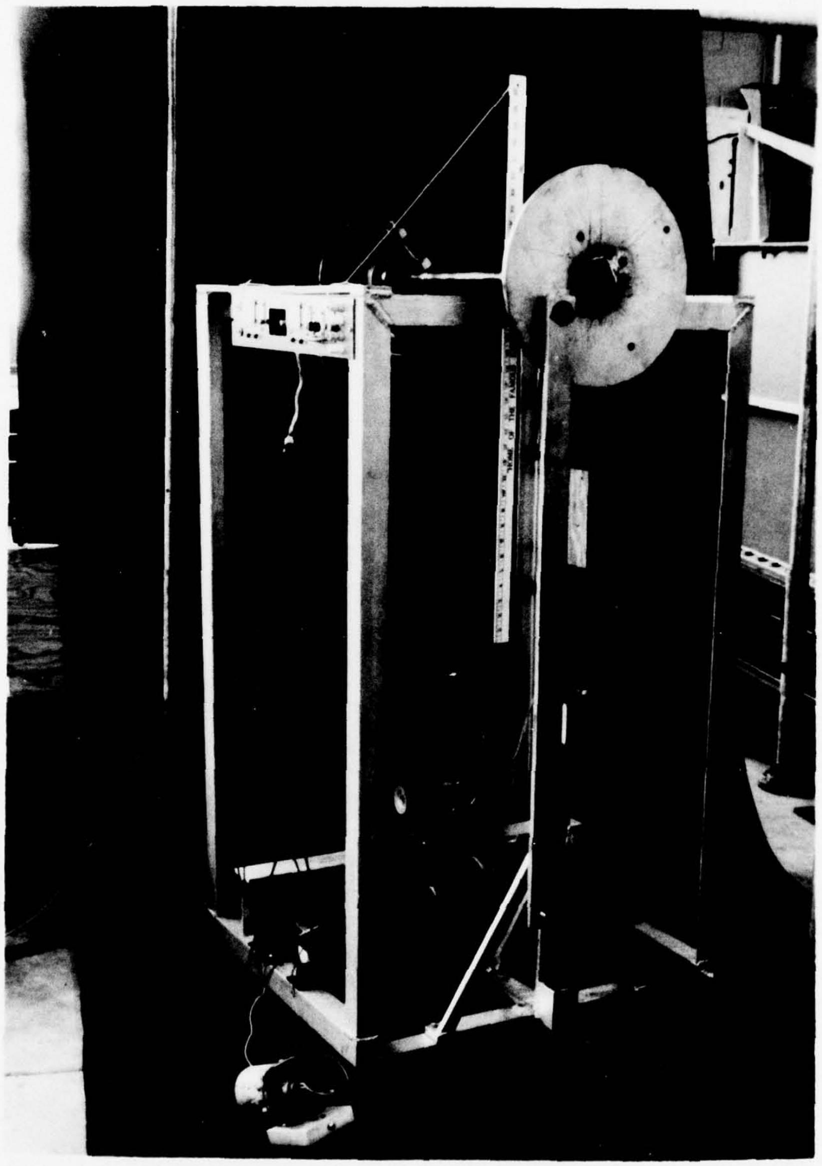
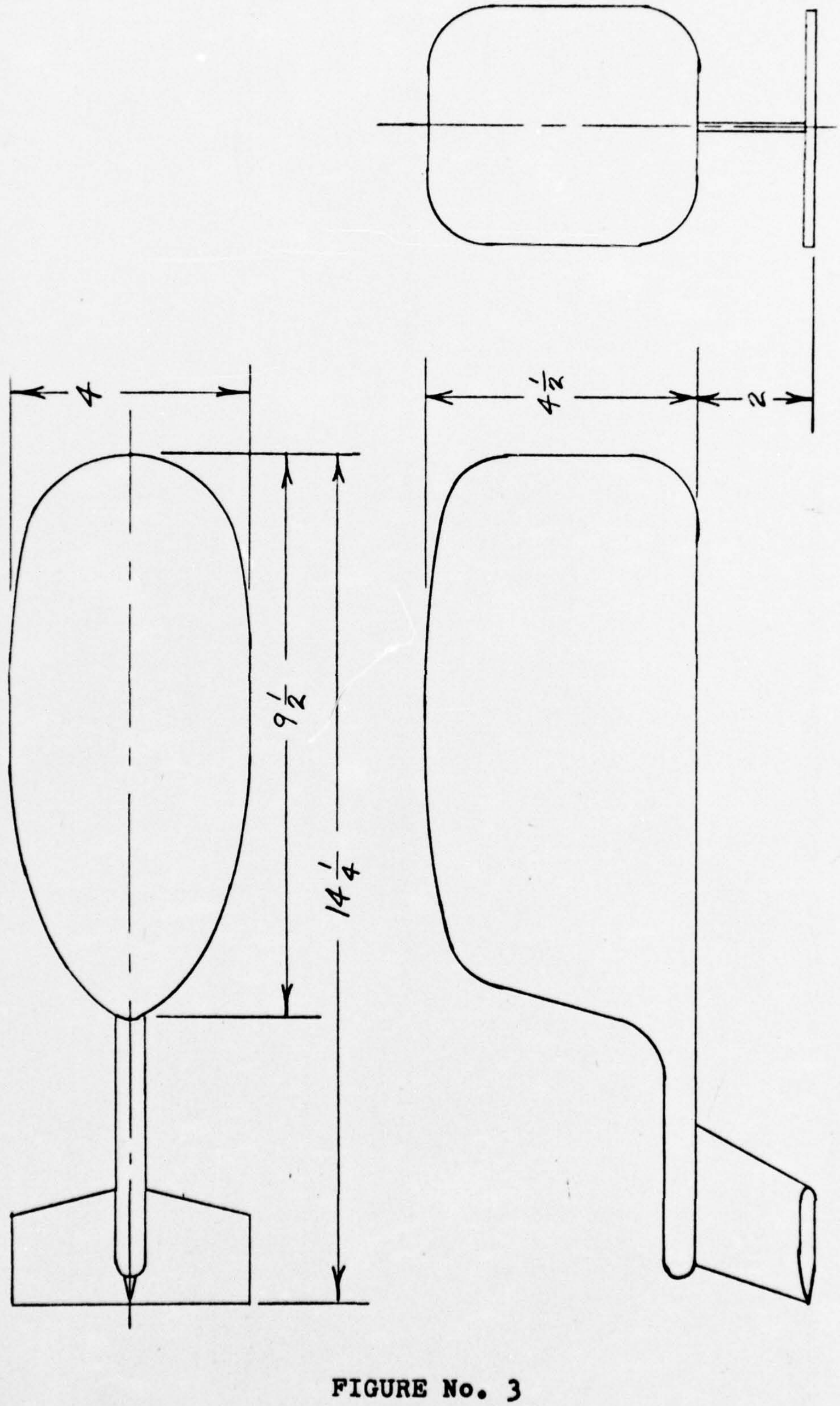


Figure 2.
Photograph of Test Apparatus



STREAMLINED BODY
(Body No. 23)

FIGURE No. 3

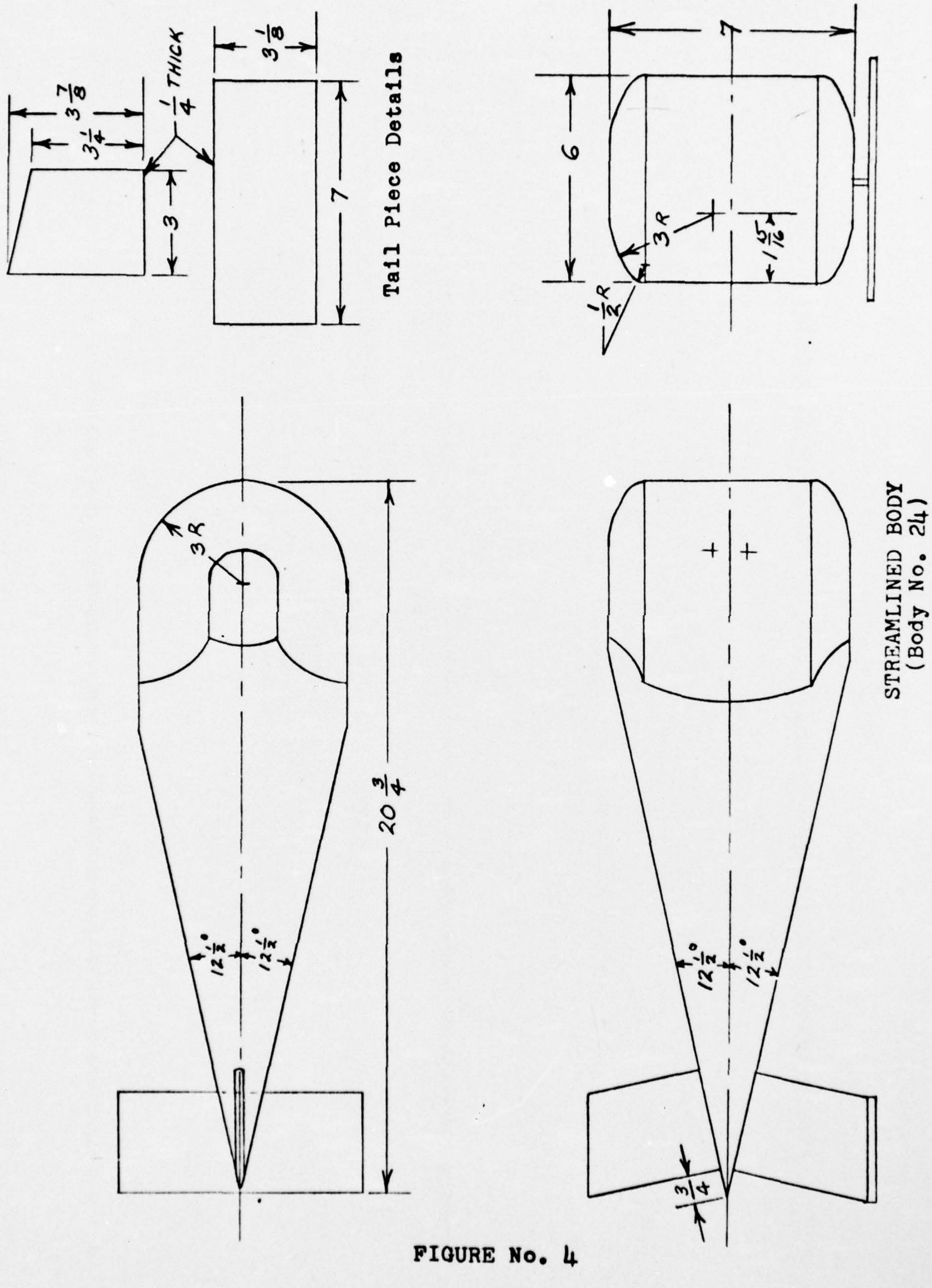

Tail Piece Details
$3\frac{7}{8}$
$3\frac{1}{4}$
$\frac{1}{4}$ THICK
3
7
$3\frac{1}{8}$
7
6
3R
$1\frac{15}{16}$
$\frac{1}{2}$R
3R
$20\frac{3}{4}$
$12\frac{1}{2}°$
$12\frac{1}{2}°$
$12\frac{1}{2}°$
$12\frac{1}{2}°$
$\frac{3}{4}$
STREAMLINED BODY
(Body No. 24)

FIGURE No. 4

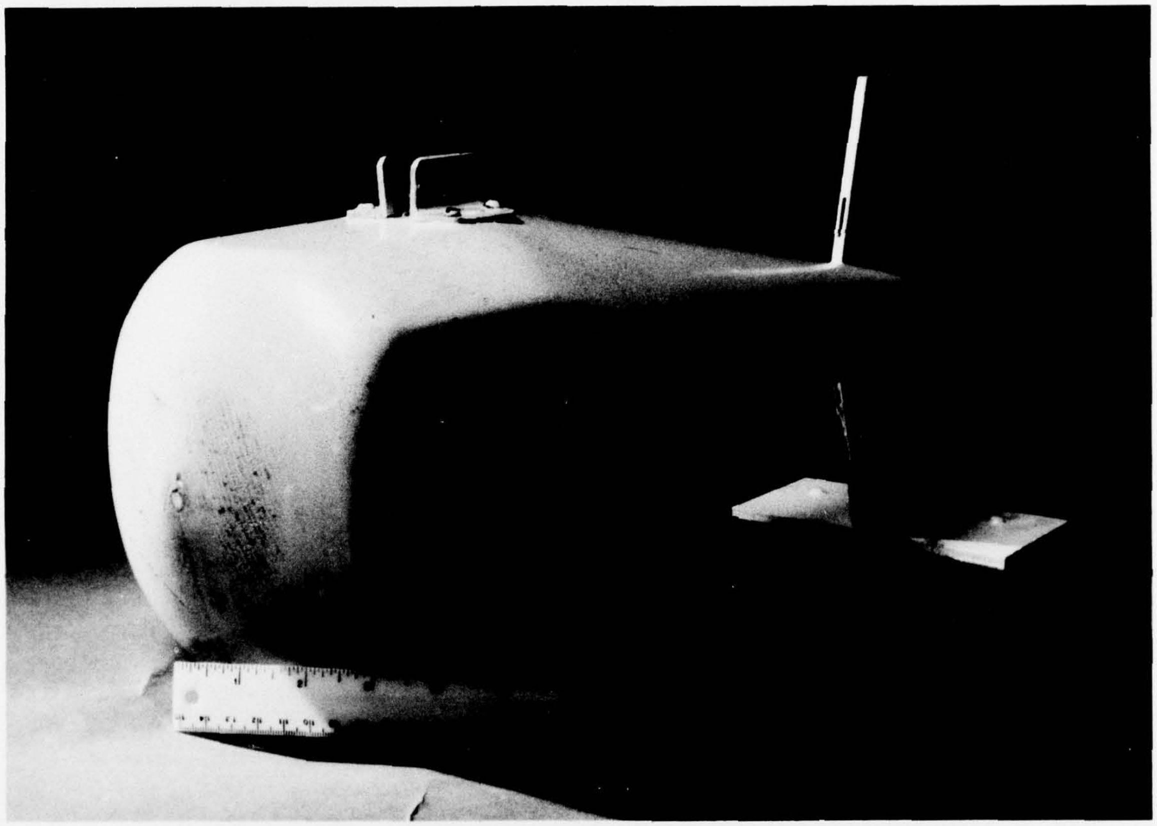
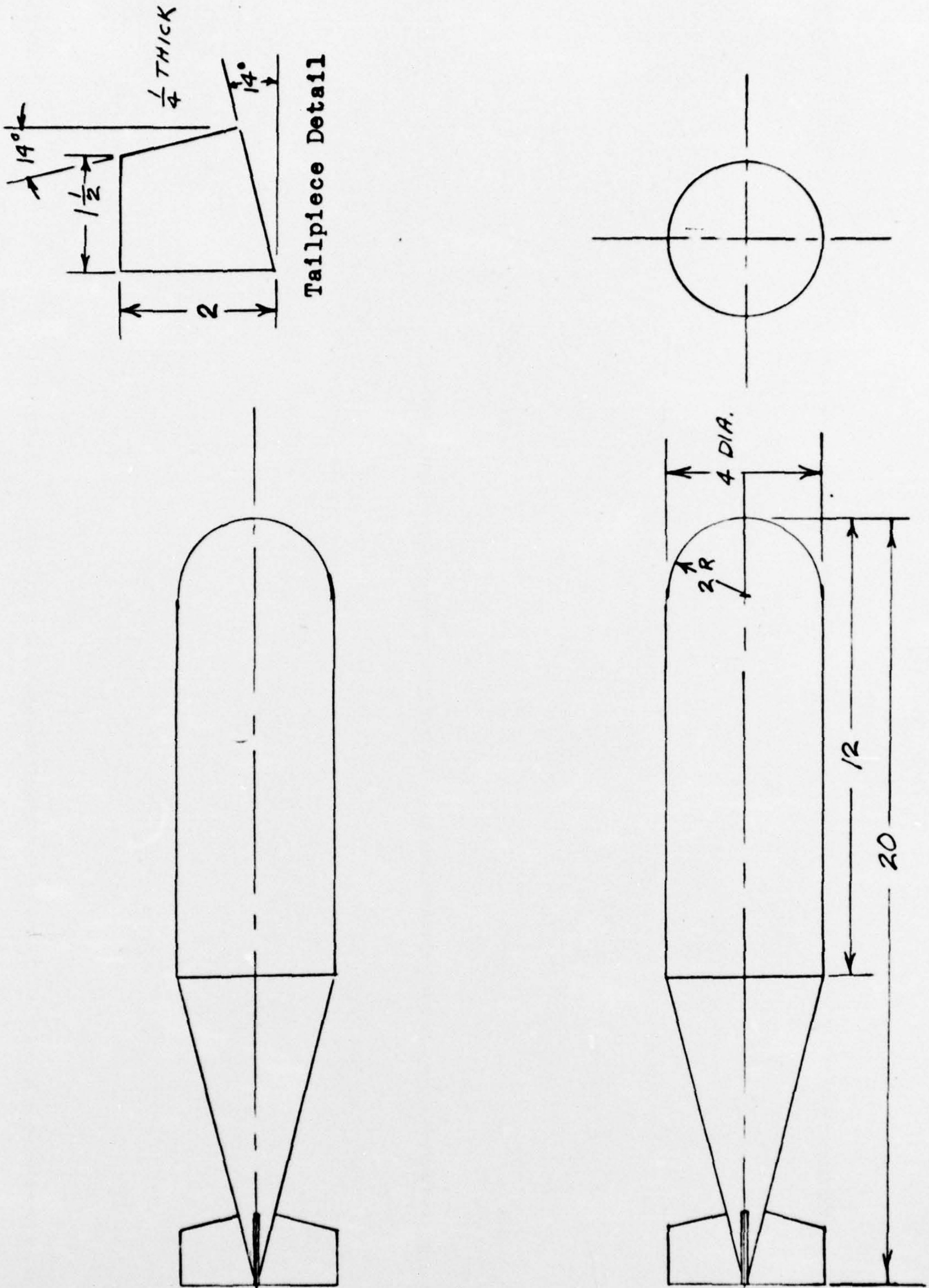


Figure 5
Streamlined Body
(Body No. 24)



$\frac{1}{4}$ THICK
14°
14°
$1\frac{1}{2}$
2
Tailpiece Detail
4 DIA.
2R
12
20
TORPEDO BODY
(Body No. 25)

FIGURE No. 6

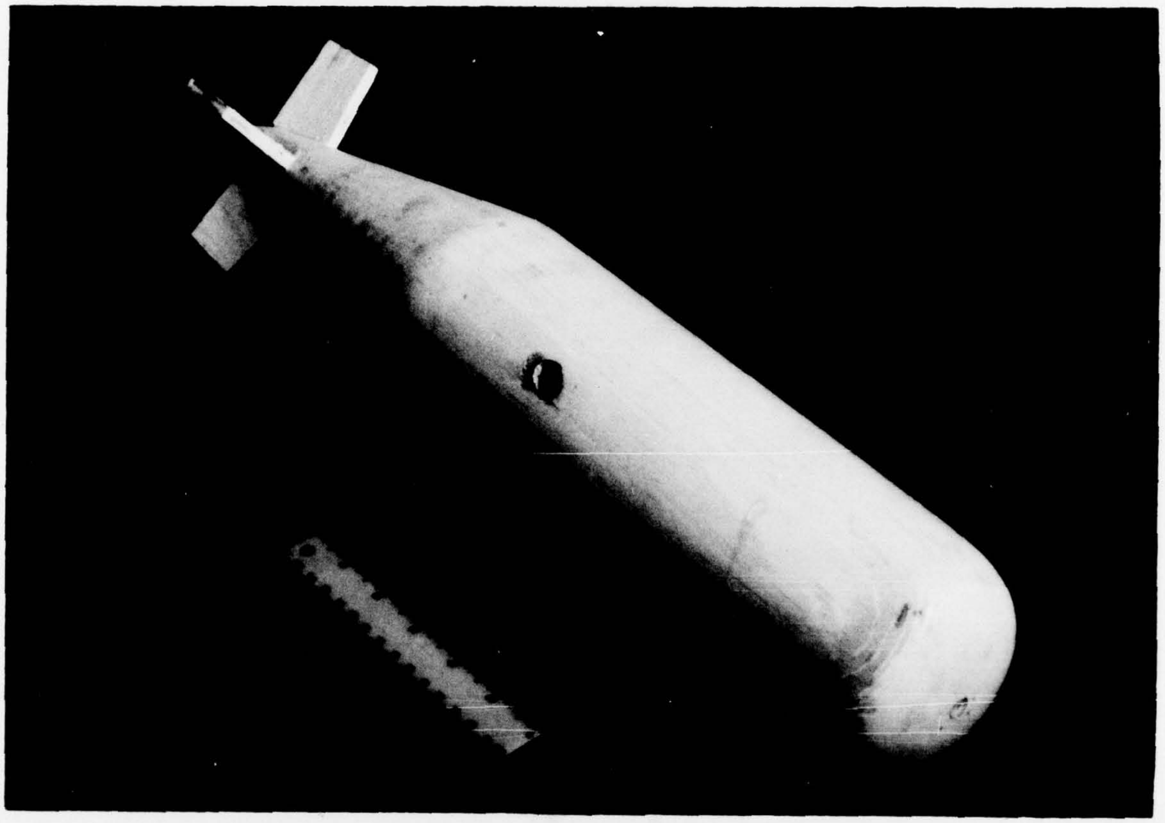
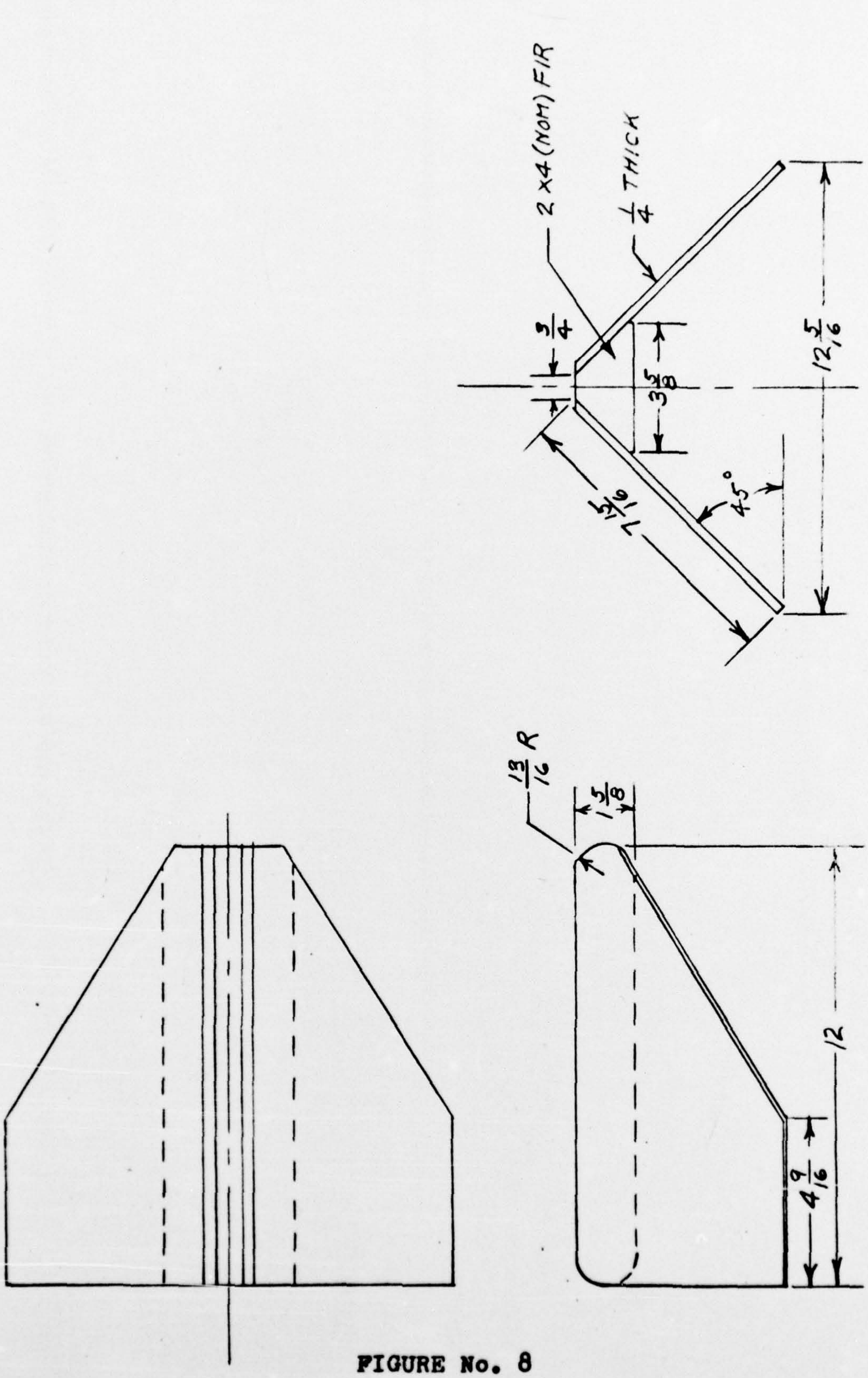


Figure 7
Torpedo Body
(Body No. 25)



V-FIN BODY
(Body No. 26)

FIGURE No. 8

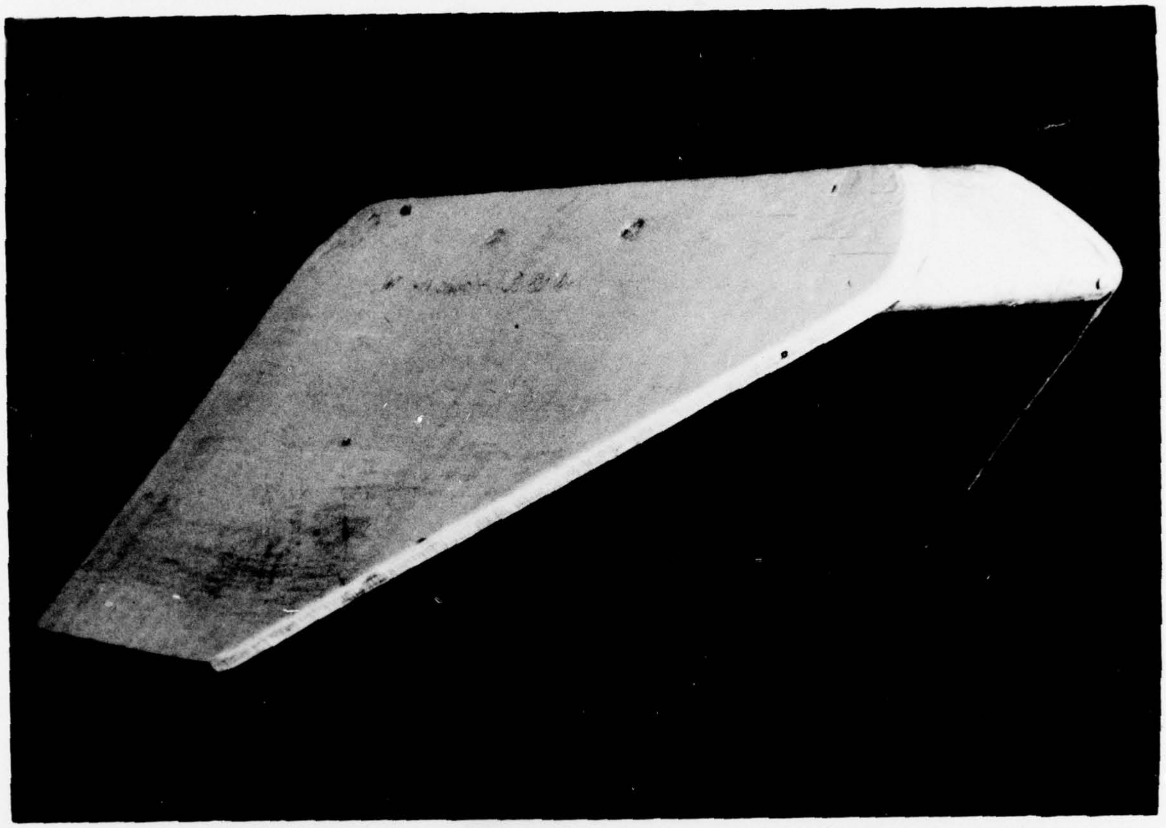


Figure 9
V-Fin Body
(Body No. 26)

APPENDIX C

TABLES

TABLE I

Description of Bodies Used in Hydrodynamic Mass Tests

Body No.	Description	Characteristic Dimensions (inches) X Horiz.	Y Horiz.	Z Vert.	Material
1	2:1 ellipsoid	24.0	12.0	12.0	soft pine
2	sphere	18.0	18.0	18.0	"
3	2:1 ellipsoid with 1" thk. pine "wings" attached to either end. Total "wing" area = 20% of area of ellipsoid section.	36.0	12.0	12.0	"
4	2:1 ellipsoid with "wings" "wing" area = 30% of ellipsoid section	36.0	12.0	12.0	"
5	2:1 ellipsoid with "wings" "wing" area = 40% of ellipsoid section	36.0	12.0	12.0	"
6	2:1 ellipsoid with "wings" "wing" area = 50% of ellipsoid section	36.0	12.0	12.0	"
7	circular disc	6.0	6.0	.25	steel
8	circular disc	12.0	12.0	.375	fir plywd.
9	square plate	10.625	10.625	.375	"
10	1.5:1 rectangular plate	13.00	8.75	.375	"
11	2:1 rectangular plate	15.00	7.50	.375	"
12	2.5:1 rectangular plate	16.75	6.75	.375	"
13	3:1 rectangular plate	18.375	6.125	.375	"
14	60°-60°-60° triangular plate	16.1875	14.0	.375	"
15	45°-90°-45° triang. plate	21.25	10.625	.375	"
16	1.5:1 elliptical disc	14.75	9.75	.375	"

TABLE I (continued)

Body No.	Description	Characteristic Dimensions (inches) X Horiz.	Y Horiz.	Z Vert.	Material
17	2:1 elliptical disc	17.0	8.5	.375	fir plywd.
18	4:1 rectangular plate	20.0	5.0	.375	"
19	5:1 rectangular plate	25.0	5.0	.375	"
20	sphere	12.0	12.0	12.0	soft pine
21	sphere - hollow, free flooding (rubber ball)	4.0	4.0	4.0	soft rubber
22	I-beam 2" wide, 3½" deep web & flange thickness 3/4"	24.0	2.0	3.5	soft pine
23	streamlined body (refer to fig. 3)	14.25	4.0	6.5	mahogany
24	streamlined body (refer to fig. 4)	20.75	6.0	7.0	soft pine
25	"torpedo" type body (refer to fig. 6)	20.0	4.0	4.0	"
26	"V"-fin type body (refer to fig. 8)	12.3125	12.0	5.625	s.pine & fir plywd.
27	cube 1:1:1	3.5	3.5	3.5	fir
28	parallelepiped 1:1:2	3.5	3.5	7.0	"
29	parallelepiped 1:1:3	3.5	3.5	10.5	"
30	parallelepiped 1:1:4	3.5	3.5	14.0	"
31	parallelepiped 1:1:5	3.5	3.5	17.5	"
32	parallelepiped 1:1:6	3.5	3.5	21.0	"
33	parallelepiped 1:1:7	3.5	3.5	24.5	"

Table 2

Hydrodynamic Mass and Mechanical Impedance
For Discs and Plates

Mean Displacement to Diameter Ratio - 0.17
Mean Dimensionless Frequency - 15.46×10^{-3}
Mean Submergence to Diameter Ratio - 2.83

Body Shape	Hydrodynamic Mass Factor	Impedance Factor	Phase Angle
Circular Disc	1.079	1.218	63.39°
1.5:1 Elliptical Disc	.841	.994	58.55°
2:1 Elliptical Disc	.922	1.080	59.30°
Square Plate	.502	.687	51.00°
1.5:1 Rectangular Plate	.912	.977	65.01°
2:1 Rectangular Plate	.826	.988	56.85°
2.5:1 Rectangular Plate	1.035	1.215	58.63°
3:1 Rectangular Plate	1.148	1.422	55.78°
4:1 Rectangular Plate	1.043	1.417	51.24°
5:1 Rectangular Plate	1.092	1.524	47.37°
60°-60°-60° Triangular Plate	1.069	1.277	57.48°
45°-90°-45° Triangular Plate	1.130	1.381	56.24°

Table 3

Hydrodynamic Mass and Mechanical Impedance For A Sphere

Mean Displacement to Diameter Ratio - 0.67
Mean Dimensionless Frequency - 15.20×10^{-3}
Mean Submergence to Diameter Ratio - 1.64

Hydrodynamic Mass Factor	Impedance Factor	Phase Angle
0.632	0.630	80.87°

Table 4

Hydrodynamic Mass and Mechanical Impedance for a 2:1 Ellipsoid With and Without Wings

Mean Displacement to Diameter Ratio - 0.42
Mean Dimensionless Frequency - 12.24×10^{-3}

I. Submergence to Diameter Ratio - 1.0

Wing Area (as a percentage of ellipse section)	Hydrodynamic Mass Factor (based on ellipsoid without wings)	Impedance Factor (based on ellipsoid without wings)	Phase Angle
0%	0.892	0.953	83.20°
20%	1.037	1.077	73.00°
30%	1.298	1.346	79.81°
40%	1.461	1.492	79.21°
50%	1.572	1.604	79.22°

II. Submergence to Diameter Ratio - ∞

0%	0.702
20%	0.815
30%	1.024
40%	1.150
50%	1.237

Table 5

Hydrodynamic Mass and Mechanical Impedance
for an I-beam Type Section

Mean Displacement to Diameter Ratio - 0.945
Mean Dimensionless Frequency - 3.93×10^{-3}
Mean Submergence to Diameter Ratio - 10.56

Hydrodynamic Mass Factor	Impedance Factor	Phase Angle
2.110	3.893	36.25°

Table 6

Hydrodynamic Mass and Mechanical Impedance
for Four Typical Towed Bodies

Body No.	Mean Displacement to Diameter Ratio	Mean Dimensionless Frequency x 10^3	Mean Submergence to Diameter Ratio	Hydrodynamic Mass Factor	Impedance Factor	Phase Angle
23	1.03	7.82	5.81	1.316	1.379	54.13°
24	0.53	10.09	3.15	0.787	0.827	72.88°
25	0.99	7.75	5.24	0.820	0.852	72.87°
26	0.15	18.14	1.69	0.429	0.456	70.12°

Table 7

Hydrodynamic Mass and Mechanical Impedance for Parallelepipeds of Square Section

Mean Displacement to Diameter Ratio - 1.46
Mean Dimensionless Frequency - 7.99×10^{-3}
Mean Submergence to Diameter Ratio - 7.54

Parallelepiped Height to Width Ratio	Hydrodynamic Mass Factor	Impedance Factor	Phase Angle
1	2.123	2.452	58.77°
2	0.848	0.960	61.38°
3	0.651	0.724	64.21°
4	0.440	0.528	54.23°
5	0.443	0.494	63.73°
6	0.289	0.399	43.96°
7	0.219	0.292	42.38°

Table 8

Results of Forced Oscillation Tests
For a Sphere

Mean Submergence to Diameter Ratio - 2.208
Mean Dimensionless Frequency - 13.06 x 10^{-3}
Displacement to Diameter Ratio - 0.333

Hydrodynamic Mass Factor	Impedance Factor	Phase Angle
0.715	1.020	75.06°

Table 9

Results of Forced Oscillation Tests
For a Circular Disc

Mean Submergence to Diameter Ratio - 2.0
Mean Dimensionless Frequency - 7.09 x 10^{-3}

Displacement to Diameter Ratio	Hydrodynamic Mass Factor	Impedance Factor	Phase Angle
0.333	1.498	3.333	68.39°
0.500	1.757		
0.666	2.059		